УДК 624.04 + 693

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STRUCTURAL RELIABILITY IN EUROCODES

The structural reliability recommended in Eurocodes and other international documents vary within a broad range, while the reference to the failure consequences and design working life is mentioned only very vaguely. In some cases the target reliability indexes are indicated for one or two reference periods (in Eurocodes for 1 year and 50 years), however no explicit link to the design working life is usually provided. This article attempts to clarify the relationship between the target reliability levels, failure consequences, the design working life and the discount rate. The theoretical study based on probabilistic optimization is supplemented by recommendations useful for code makers and required by practicing engineers. It appears that the optimum reliability indices depend primarily on the ratio of the cost of structural failure to the cost per unit of structural parameter, and less significantly on the design working life and on the discount rate.

Key words: structural reliability, probabilistic model, design working life, reliability margin, basic variables.

Introduction. The target reliability levels recommended in various national and international documents are inconsistent in terms of the recommended values and the criteria for their selection. In general, optimum reliability levels can be obtained by considering both the cost of the structure and the expected cost of failure over the design working life.

The design working life is understood as an assumed period of time for which a structure is to be used for its intended purpose without any major repair being necessary. Indicative values of design working life (10 to 100 years for different types of structures) are given in EN 1990 (2002). Recommended values of reliability indexes are given for two reference periods 1 year and 50 years (see Table 1), without any explicit link to the design working life that generally differs from the reference period.

It should be emphasized that the reference period is understood as a chosen period of time used as a basis for statistically assessing time variable basic random variables and the corresponding probability of failure. The concept of reference period is therefore fundamentally different form the concept of design working life. Confusion is often caused when the difference between these two concepts is not recognized.

It should be recognized that the couple of β values (for 1 year and 50 years) given in Table 1 for each reliability class correspond to the same reliability level. Practical application of these values, however, depends on the time period considered in the verification, which may be linked to available probabilistic information concerning time variant basic variables (imposed load, wind, earthquake, etc.).

Table 1. Reliability classification in accordance with EN 1990 (2002)

Reliability classes	Consequences of structural failure	Reliability index β for reference period		Examples of buildings and civil engineering works
		1 year	50 years	
RC3 – high	High	5,2	4,3	Bridges, public buildings
RC2 – normal	Medium	4,7	3,8	Residences and offices
RC1 – low	Low	4,2	3,3	Agricultural buildings

For example, considering a structure of reliability class 2 having a design working life of 50 years, the reliability index $\beta = 3{,}8$ should be used provided that probabilistic models of basic variables are available for this period. The same reliability level is achieved when a reference period of 1 year and a target of β = 4,7 are applied using the theoretical models for a reference period of one year.

A more detailed recommendation concerning the target reliability is provided by ISO 2394 (1998) where the target reliability indexes are indicated for the whole design working life without any restriction and related not only to the consequences but also to the relative costs of safety measures (see Table 2).

Table 2. Examples of life-time target reliability indexes β in accordance with ISO 2394 (1998)

Relative costs of safety measures	Consequences of failure			
	Small	Some	Moderate	Great
High	0	1,5	2,3	3,1
Moderate	1,3	2,3	3,1	3,8
Low	2,3	3,1	3,8	4,3

Similar recommendations are provided in the JCSS (2001) Probabilistic model code (Table 3). Recommended target reliability indexes are also related to both the consequences and to the relative costs of safety measures, however for the reference period 1 year. The consequences classes in ICSS (2001) (similar to those in EN 1990, 2002) are linked to the ratio between the total cost (cost of construction plus direct failure costs) to the construction cost as follows:

Both documents ISO 2394 (1998) and JCSS (2001) seem to recommend reliability indexes that are lower than those given in EN 1990 (2002) even for the “small relative costs” of safety measures. It should be noted that EN 1990 (2002) gives the reliability indexes for two reference periods 1 and 50 years that may be accepted as the design working life for common structures (see also a discussion provided by Diamantidis, 2009). ISO 2394 (1998) recommends indexes for “life-time, examples”, thus related to the design working life and Probabilistic Model Code by JCSS (2001) provides reliability indexes for the reference period of 1 year.

Table 3. Tentative target reliability indexes β (and associated target failure rates) related to one year reference period and ultimate limit states in accordance with JCSS (2001)

Relative costs of safety measures	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large	$\beta = 3{,}1$ ($p \approx 10^{-3}$)	$\beta = 3{,}3$ ($p \approx 5\times10^{-4}$)	$\beta = 3{,}7$ ($p \approx 10^{-4}$)
Normal	$\beta = 3{,}7$ ($p \approx 10^{-4}$)	$\beta = 4{,}2$ ($p \approx 10^{-5}$)	$\beta = 4{,}4$ ($p \approx 5\times10^{-6}$)
Small	$\beta = 4{,}2$ ($p \approx 10^{-5}$)	$\beta = 4{,}4$ ($p \approx 5\times10^{-6}$)	$\beta = 4{,}7$ ($p \approx 10^{-6}$)

However, a clear link between the design working life and the target reliability level is not apparent from any of the above mentioned documents. Thus, it is not clear which target reliability index should be used for a given design working life different from 50 years (say 10 years). The basic aim of this contribution is to clarify the link between the design working life and the reliability index and to provide guidance for specification of the target reliability level for a given design working life. The submitted theoretical study based on probabilistic optimization is supplemented by practical recommendations.

General principles of probabilistic optimization. Probabilistic optimization is based on a fundamental form of the objective function expressed as the present value of the total cost $C_{tot}(x,q,n)$

$$C_{\mathrm{tot}}(x,q,n) = C_f \sum_{i=1}^{n} P_f(x,i)\,Q(q,i) + C_0 + xC_1, \quad (1)$$

where x *is* the decision parameter of the optimization (e.g. a parameter of structural resistance), q is the annual discount rate (e.g. 0,03, an average long run value of the real annual discount rate in European countries), n is the number of years of a considered design working life (e.g. 50, 100), $P_f(x,i)$ is the failure probability in year i, C_f is the malfunctioning cost (due to the loss of structural functionality), $Q(q,i)$ is the discount factor dependent on the annual discount rate q and the number of years i, C_0 is the initial cost independent of the decision parameter x, and C_1 is the cost per unit of the decision parameter x.

Note that the design working life is considered here as a given deterministic quantity characterized by the number of years n. In reality the working life for a given design may be a random quantity depending on social and physical factors. The design itself may aim at some optimum. This option of random design working life is, however, neglected in this study.

Assuming almost independent failure events in subsequent years, the annual probability of failure $P_f(x,i)$ at the year i is given by the geometric sequence

$$P_f(x,i) = p(x)\,(1 - p(x))^{i-1}. \tag{2}$$

The initial probability of failure $p(x)$ is dependent on the decision parameter of structural resistance x. Note that annual failure probabilities can be assumed to be independent when failure probabilities are chiefly influenced by time-variant loads (climatic actions, traffic loads, accidental loads). Then the failure probability $P_{fn}(x)$ during n years can be estimated by the sum of the sequence $P_f(x,i)$ given as

$$P_{fn}(x,n) = p(x)\,(1 - p(x))^n \approx n\,p(x). \tag{3}$$

Note that the approximation indicated in equation (3) is acceptable for small probabilities $p(x) < 10^{-3}$.

The discount factor of the present value of the expected future costs at the year i is considered in the usual form as

$$Q(q,i) = 1 / (1+q)^i. \tag{4}$$

Thus, the cost of malfunctioning C_f is discounted by the factor $Q(q,i)$ depending on the discount rate q and the point in time (number of year i) when the loss of structural utility occurs. Considering equations (2) and (4) the total costs $C_{tot}(x,q,n)$ described by equation (1) may be written as

$$C_{tot}(x,q,n) = C_f\,p(x)\,PQ\,(x,q,n) + C_0 + x\,C_1. \tag{5}$$

Here the total sum of expected malfunction costs during the period of n years is dependent on the product of the present value of malfunction cost C_f, the annual probability $p(x)$ and a sum of the geometric sequence having the quotient $(1- p(x))/(1+ q)$, denoted as the time factor $PQ(x,q,n)$:

$$PQ\left(x,q,n\right) = \frac{1 - \left[\dfrac{1-p(x)}{1+q}\right]^n}{\dfrac{1-p(x)}{1+q}}. \tag{6}$$

In general the total cost $C_{tot}(x,q,n)$ depends on the costs C_0, C_1, C_f, the annual probability of failure $p(x)$, the discount rate q and the design working life n. Note that for small probabilities of failure $p(x)$ (for appropriate structural parameter x) and small discount rate q, the time factor $PQ(x,q,n) \approx n$.

The necessary condition for the minimum of the total cost follows from (1) as

$$\frac{\partial C_{\text{tot}}(x,q,n)}{\partial x} = C_f \sum_{i=1}^{n} Q(q,i) \left[\frac{\partial P_f(x,i)}{\partial x}\right]_{x=x_{\text{opt}}} + C_1 = 0. \tag{7}$$

Thus, the optimum structural parameter may be determined from the equation

$$\sum_{i=1}^{n} Q(q,i) \left[\frac{\partial P_f(x,i)}{\partial x}\right]_{x=x_{\text{opt}}} = -\frac{C_1}{C_f}. \tag{8}$$

Equation (8) represents a general form of the necessary condition for the minimum of total cost $C_{tot}(x,q,n)$ and the optimum value x_{opt} of the parameter x and the optimum annual probability of failure $p_{opt} = p(x_{opt})$. The optimum probability for the total design working life $T_d = n$ years follows from equation (2) as

$$P_{fn,opt} = 1 - (1 - p_{opt})^n \approx n\, p_{opt}. \tag{9}$$

The corresponding optimum reliability index $\beta_{opt} = -\Phi^{-1}(P_{fn,opt})$. These quantities are in general dependent on the cost ratio C_f/C_1, discount rate q and the design working life n.

Failure probability of a generic structural member

Consider a generic structural member described by the limit state function $Z(x)$

$$Z(x) = x\,f - (G+Q). \tag{10}$$

Here x denotes a deterministic structural parameter (e.g. the cross-section area), f the strength of the material, G the appropriate load effect due to permanent load and Q the load effect due to variable load. Theoretical models of the random quantities f, G and Q considered in the following example are given in Table 4.

Table 4. Theoretical models of the random variables f, G and Q (annual extremes)

Variables	Distribution	The mean	Standard deviation	Coef. of variation
f	Lognormal	100	10	0,10
G	Normal	35	3,5	0,10
Q	Gumbel	10	5	0,50

Considering the theoretical models given in Table 4, the reliability margin $Z(x)$ may be well approximated by the three parameter lognormal distribution $\Phi_{Z(x)}$. The annual failure probability $p(x)$ is then given as

$$p(x) = \Phi_{Z(x)}(Z(x) = 0). \tag{11}$$

In equation (11) the three-parameter lognormal distribution is evaluated for $Z(x) = 0$.

An example. The following example illustrates the general principles and a special case of probabilistic optimization. To simplify the analysis the total costs $C_{tot}(x,q,n)$ given by equation (1) are transformed to the standardized form $\kappa_{tot}(x,q,n)$ given as

$$\kappa_{tot}(x,q,n) = \frac{C_{tot}(x,q,n) - C_0}{C_1} = \frac{p(x)PQ(x,q,n)C_f}{C_1} + x. \tag{12}$$

Obviously, both the total costs $C_{tot}(x,q,n)$ and the total standardized cost $\kappa_{tot}(x,q,n)$ achieve the minimum for the same structural parameter x_{opt}. The annual probability of failure $p(x)$ considered here for a general structural member is given by equation (11). However, the following procedure may be applied for any relevant dependence of the failure probability $p(x)$ expressed as a function of a suitable structural parameter x.

In the example shown in Figure 1 it is assumed that the discount rate is $q = 0{,}03$ and the design working life is $n = 50$ years. Under these assumptions, Figure 1 shows the variation of the total standardized costs $\kappa_{tot}(x,q,n)$ (given by equation (12)), and the optimum reliability index β_{opt}, with structural parameter x. The optimal values $x_{opt}(q,n)$ of the structural parameter x, given by equation (8), are indicated by the dotted vertical lines.

The optimal reliability index. In general, the optimal reliability index $\beta_{opt}(q,n,C_f/C_1)$ depends on the discount rate q, the design working life n and the cost ratio C_f/C_1. However, the index β_{opt} is primarily dependent on the cost ratio C_f/C_1, and its dependence on the discount rate q and the design working life n seems to be insignificant. This is well illustrated by Figure 2 which shows the variation of the optimal reliability index β_{opt} with the cost ratio C_f/C_1 for selected design working life $n = 1$, 50, 100, and the discount rate $q = 0{,}03$.

Figure 3 shows variation of the optimum reliability index β_{opt} with the cost ratio C_f/C_1 for discount rates $q = 0{,}01$, 0,03 and 0,05 and for the design working life $n = 50$. It follows from Figure 2 and 3 that the optimal reliability index β_{opt} decreases slightly with increasing design working life n and increasing discount rate q.

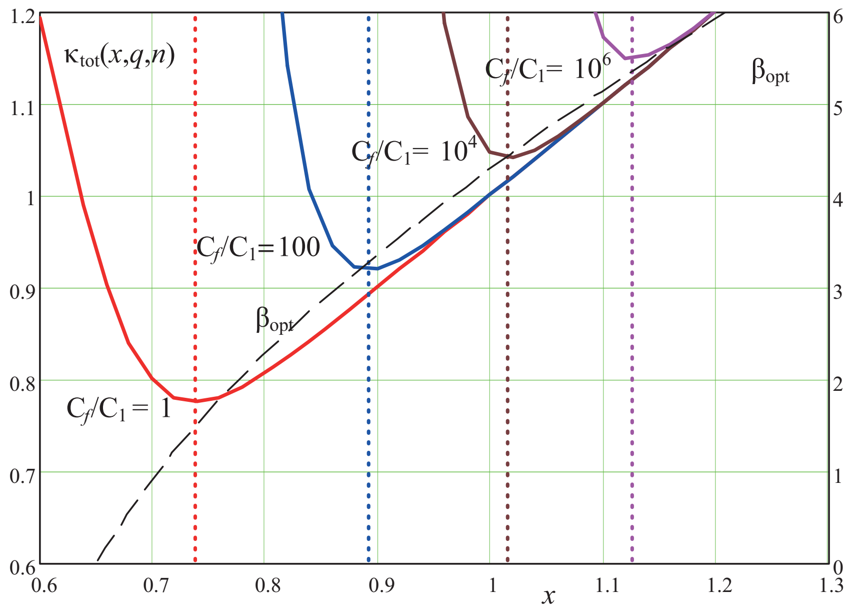

Fig. 1. Variation of the total standardized cost $\kappa_{tot}(x,q,n)$ and the optimum reliability index β_{opt} with the parameter x for q =0,03, n = 50 and selected cost ratios C_f/C_1

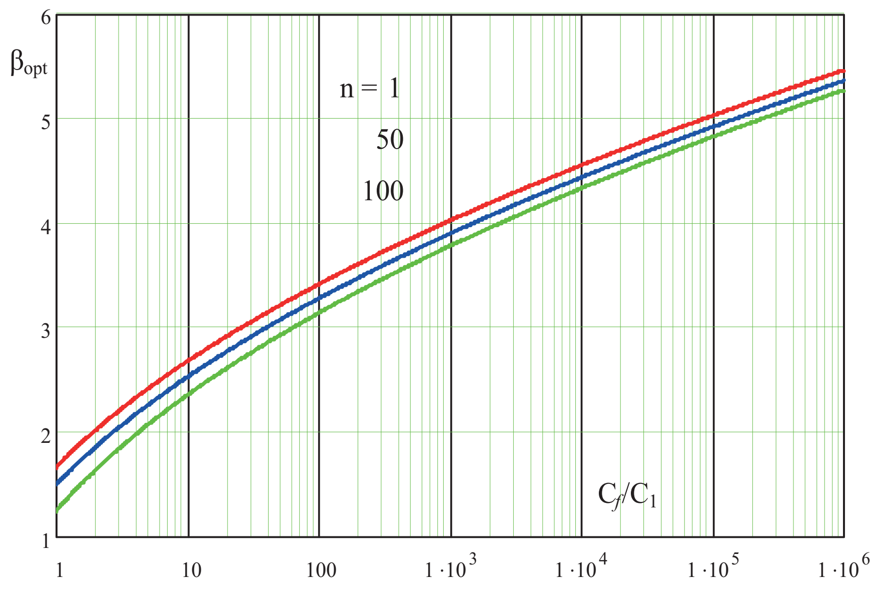

Fig. 2. Variation of the optimum reliability index β_{opt} with the cost ratio C_f/C_1 for selected design working life n = 1, 50, 100, and the discount rate q = 0,03

Figure 3 also shows the target reliability indexes recommended by ISO 2394 (1998) and JCSS (2001) respectively. The range of life time target reliability levels recommended by ISO 2394 for various combinations of consequences of failure (related to C_f) and of safety measures (related to C_1) corresponds to the range of C_f/C_1 between 1 and 10^4 whilst the value recommended by JCSS (2001) extends towards 10^5. Note that the significant differences between one and fifty year reference periods indicated by EN 1990 (2002) in Table 1 appear from Figure 5 to be less severe, due to the effect of discounting of future failures.

The upper range of optimal reliability levels ($\beta_{opt} > 5$) are not covered by recommended target reliability values indicated in documents EN 1990 (2002), ISO 2394 (1998) and JCSS (2001) and shown in Table 1, 2 and 3. The upper range of the reliability levels corresponds, however, to implicit reliability levels of presently valid structural design standards (Holicky and Schneider 2001).

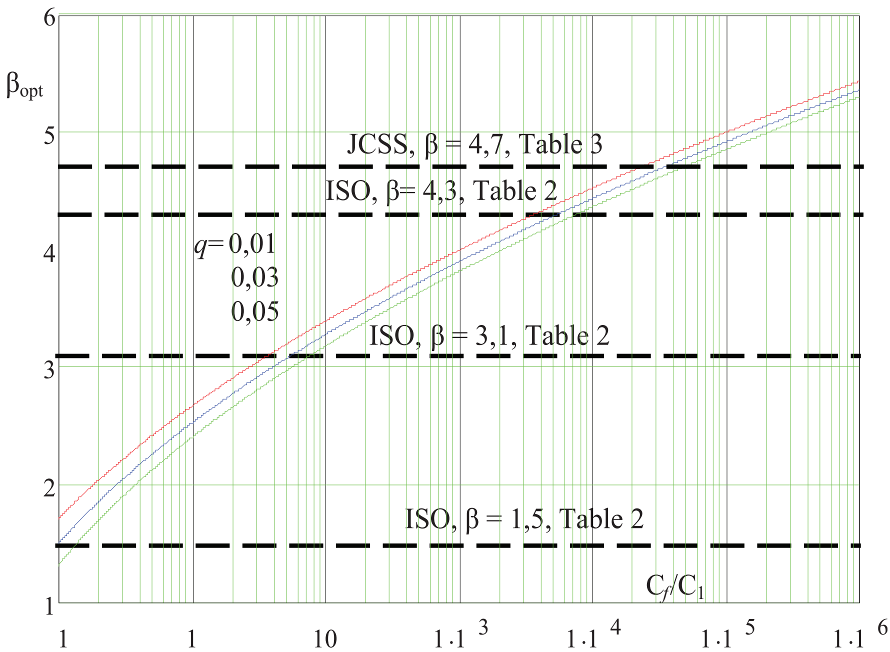

Fig. 3. Variation of the optimum reliability index β_{opt} with the cost ratio C_f/C_1 for selected discount rates q = 0,01, 0,03, 0,05, the design working life n = 50 and target reliability levels in accordance with ISO 2394 (1998) and JCSS (2001)

It should be mentioned that in some cases the target reliability index β_t can differ from the optimal reliability index β_{opt}, for example when the cost ratio C_f/C_1 is unknown or difficult to assess. Then a conservative value assessed for reasonable lower bounds of the design working life (say 50 years) and the discount rate (say 0,02) can be used.

Conclusions and recommendations. Available documents and codes for structural design do not provide a clear link between the design working life and the target reliability level and no recommendations are offered to specify the target reliability index for temporary structures having a short design working life. It is shown that the target reliability of structures can be derived from theoretical principles of probabilistic optimization providing an indication of the relative significance of the various influencing factors, particularly of the design working life, the discount rate and of the respective costs of the loss of structural utility versus the incremental cost of adjusting structural resistance.

An example of probabilistic optimization clearly shows that the optimal probability of failure and the reliability indexes generally depend on:

1. the ratio of the cost of structural failure to the cost per unit of structural parameter,
2. the design working life,
3. the discount rate.

The obtained results farther indicate more specific conclusions, the validity of which should be conditional on the assumptions concerning the objective function and on the annual failure probability. It appears that with increasing malfunctioning cost, the optimal reliability index and the optimal structural parameter increase (Figure 1). The design working life seems to have a very limited influence on the optimal reliability (Figure 2). Even less significant seems to be effect of the discount rate (Figure 3) particularly for temporary structures.

For practical purposes the optimal target reliability index and the corresponding structural parameter can be well assessed considering relevant lower bounds for the design working life and the discount rate. The results can be implemented in the partial factor method as follows:

• the characteristic values of the basic variables including time varying loads (wind, snow etc) may remain independent of the design working life;

• the design values are specified on the bases of appropriate reliability index assessed for given cost ratio, design working life and discount rate

• the partial factors are determined considering specified design values and unchanged characteristic values of basic variables.

It should be stated that further investigations are planned to analyze the important aspects of reliability differentiation taking into account consequences, the design working life and the discount rate and to illustrate implementation of achieved results in practical design.

Acknowledgements. This study is an outcome of the research project GAČR P105/12/0589: "Probabilistic optimization of the target structural reliability".

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Поступила в редакцию в октябре 2012 г.

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For citation: Holický M. Structural Reliability in Eurocodes. *Vestnik MGSU* [Proceedings of Moscow State University of Civil Engineering]. 2012, no. 11, pp. 117—124.

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НАДЕЖНОСТЬ КОНСТРУКЦИЙ В СООТВЕТСТВИИ С ЕВРОКОДАМИ

Рекомендованные Еврокодами и прочими международными документами показатели надежности конструкций характеризуются весьма широким диапазоном значений, но последствия обрушения и проектируемая долговечность конструкций упоминается в них лишь в самых общих чертах. В некоторых случаях заданные показатели надежности указываются для одного или двух сроков (в Еврокодах указаны сроки, составляющие 1 год и 50 лет), но как правило, отсутствует четкая связь между сроками и проектируемой долговечностью конструкций. Предпринята попытка прояснить взаимосвязь между заданными уровнями надежности, последствиями обрушения конструкций, проектируемой долговечностью конструкций и корректировочными коэффициентами. Теоретическое исследование проведено на основании вероятностной оптимизации с учетом рекомендаций, которые могут представлять интерес для разработчиков нормативной документации и инженеров-практиков. Выяснено, что оптимальные показатели надежности зависят, прежде всего, от отношения стоимости последствий обрушения конструкций к удельной стоимости параметров конструкций, и в меньшей степени — от проектируемой долговечности и корректировочных коэффициентов.

Ключевые слова: надежность конструкции, вероятностная модель, проектируемая долговечность, запас долговечности, базовые переменные.

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Для цитирования: *Holický M.* Structural reliability in Eurocodes // Вестник МГСУ. 2012. № 11. С. 117—124.