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Water Flow Parameters on the Symmetry Axis and Extreme Line of Current

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ABSTRACT

Introduction. An analysis of mathematical models of two-dimensional planned flows was carried out. Such flows are characterized by local depth-averaged velocities and local depths at each point of the flow. The mathematical model formation of the water flow is based on its division into several sections. There is a section where the flow parameters (velocity, depth, width) are kept constant at the stage of flow exit from the pipe — the inertial front. The purpose of the article and its relevance are defined.
Materials and methods. By introducing dimensionless complexes on the basis of π-theorem, the formula for the length of inertial front of the water flow at its spreading from a rectangular pipe into a wide diverting channel is derived. An analogy from gas dynamics is used, namely, the transition to the plane of the velocity hodograph. Using the velocity hodograph, the distribution of depths and velocities of the flow along its longitudinal axis of symmetry and along the extreme line of current was obtained. The main computation tasks for the flow parameters have been formulated.
Results. Numerical calculations of the formulated main tasks for determining flow parameters are described. Comparison with experimental data is given and the adequacy of the refined mathematical model of a two-dimensional planned flow is confirmed.
Conclusions. The resulting formula for the length of the inertial front makes it possible to achieve the desired error in calculating the parameters of the water flow. With flow expansions up to 5, the relative error of the ordinates and flow velocities does not exceed 7–10 %. Calculation formulas and implemented programs will allow HTS designers to quickly and accurately determine the boundaries, speed and depth of free flow on the culvert.

KEYWORDS: mathematical model, hydrodynamics analysis, two-dimensional water flow, open-channel hydraulics, analytical solution

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Параметры водного потока на оси симметрии и крайней линии тока

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АННОТАЦИЯ

Введение. Проведен анализ математических моделей двумерных плановых потоков. Такие потоки характеризуются местными осредненными скоростями по глубине и местными глубинами в каждой точке потока. Формирование математической модели водного потока основано на его разделении на несколько участков. На этапе выхода потока из трубы имеется участок, где сохраняются постоянными параметры потока (скорость, глубина, ширина) — инерционный фронт. Определена цель статьи и ее актуальность.
Материалы и методы. Введением безразмерных комплексов и на основе π-теоремы выведена формула длины инерционного фронта водного потока при его растекании из прямоугольной трубы в широкое отводящее русло. Использована аналогия из газовой динамики, а именно переход в плоскость годографа скорости потока. С использованием годографа скорости получены законы распределения глубин и скоростей потока вдоль его продольной оси симметрии и вдоль крайней линии тока. Сформулированы основные задачи расчета параметров потока.
Результаты. Описаны числовые расчеты сформулированных основных задач определения параметров потока.

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Приводится сравнение с экспериментальными данными и подтверждается адекватность уточненной математической модели двумерного планового потока.

Выводы. Полученная формула длины инерционного фронта позволяет добиться желаемой погрешности расчета параметров водного потока. При расширениях потока до 5 относительная погрешность ординат и скоростей потока не превышает 7–10 %. Расчетные формулы и реализованные программы позволят проектировщикам ГТС быстрее и точнее определить границы, скорость и глубину безнапорного потока над водопропускной трубой.

КЛЮЧЕВЫЕ СЛОВА: математическая модель, гидродинамический анализ, двумерный поток воды, гидравлика открытого канала, аналитическое решение

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INTRODUCTION

The subject of research is open stationary water flows behind an unpressurised opening of rectangular cross-section, with a small vertical velocity component, namely, two-dimensional planar flows. The theory and methods for solving problems of planar hydraulics are most fully described in monographs by G.I. Sukhomel, I.I. Levi and I.A. Sherenkov [1], B.T. Emtsev [2] and V.N. Kokhanenko [3].

Often, when designing hydraulic structures, it is necessary to know the characteristics of open water flows, for which the Froude criterion is greater than one. These streams are: with free spreading of the flow from open channels or free-flow pipes into a wide channel; during the flow of flows in the connections of channels of different widths; in open spillways behind hydroelectric power plants; in case of river floods behind culvert road structures; behind small bridges. According to the nature of the flow of a turbulent water flow through a non-pressure pipe, non-pressure and semi-pressure flow regimes are considered.

Open water flow in a suddenly opening channel can have velocities substantially exceeding the permissible velocities for the unstrengthened part of the channel. As shown by field surveys conducted by the authors [1, 4], the main cause of destruction of hydraulic structures is dangerous erosion of the downstream. Therefore, to calculate the anchorage of the outlet channel, hydraulic engineering structures designers need information on the flow parameters in the vicinity of the outlet edge of the pipe. Therefore, the relevance of this work is confirmed.

A water flow model behind an unconfined outlet has several sections [3–8]. In works [3–21] the parameters and shape of free-diffusion region of turbulent flow at its outflow from unpressurized pipe into wide diversionary channel is determined experimentally and analytically. In [22, 23] a formula for inertial front length of flow at the tube outlet is given. However, there is no sufficient justification for this formula.

The purpose of this paper is to derive the formula for the inertial front length on the basis of dimension theory and π-theorem, as well as testing its appropriateness in calculating the parameters of water flows.

MATERIALS AND METHODS

There are basic units of measurement in any system of units. They are introduced from experience with the help of standards. In the SI, for example, the basic units are the meter, the second, and the kilogramme. The expression of an arbitrary unit of measurement in the basic units is called the dimensionality. For each basic unit, a notation is introduced: L is length, T is time, M is mass, etc. The theory of dimensionality proves [24, 25], that the dimension of any quantity is a degree monomial of the form $[N] = L^l \cdot T^t \cdot M^m$ ... and is called the dimension formula. The statement sought follows from the fact that the ratio of two numerical values of a physical quantity should not depend on the choice of scales for the basic units of measurement.

π*-theorem.* A relation, independent of the choice of units, between n dimensional quantities, k of which have independent dimensions, can be represented as a relation between (n–k) quantities which are dimensionless combinations of n dimensional quantities.

Formula for the length of the inertial front as it flows from a rectangular pipe into a wide diversion channel. Let us consider the problem of deriving a formula for the length of the inertial front X_D^I. The need to derive the formula is justified by the fact that along the inertial front all flow parameters retain their values. Hence, the formulas [3, 5] for calculating the flow parameters will change.

The process of free flowing of supercritical unconfined potential flow into a wide diversionary channel is determined by four parameters with:

- culvert width b, cm;
- depth of flow in the pipe h_0, cm;
- initial velocity of the flow V_0, cm/s;
- free fall acceleration $g = 981$, cm/s^2.

We introduce dimensionless complexes characterizing the spreading process:

$$\frac{X_D}{h_0};\ \frac{h_0}{b};\ \frac{h_0}{H_0},\ F_0 = \frac{V_0^2}{gh_0},\ \theta_{max},$$

where

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$$H_0 = \frac{V_0^2}{2g} + h_0 \qquad (1)$$

the constant in D. Bernoulli's integral, θ (radian) is an angle characterizing the slope of the velocity vector to the longitudinal axis of flow symmetry; $F_0 = 2\tau_0/(1-\tau_0)$. Froude's criterion and the square of the velocity coefficient $\tau_0 = V_0^2/2gH_0$, θ_{max} is the maximum flow angle.

Thus, the number of independent units is $k = 2$, the dimensional quantities $n = 4$. The number of independent dimensionless complexes is $n - k = 2$, and there is a single equation linking these complexes.

To derive the structure of the formula X_D^I, we use formula (1) in the form:

$$H_0 = h_0\left[\frac{V_0^2}{2gh_0} + 1\right] \Rightarrow \frac{H_0}{h_0} = \frac{F_0}{2} + 1 \Rightarrow$$

$$\Rightarrow \frac{h_0}{H_0} = \frac{2}{F_0 + 2}.$$

Assume that X_D^I is proportional to X*. From Figure shows that:

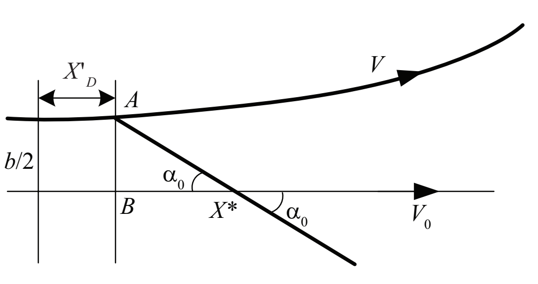
$$\text{tg}\alpha_0 = \frac{b/2}{X^*},$$

where we get

$$X^* = \frac{b/2}{\text{tg}\alpha_0},$$

where α_0 is the wave angle [3] defined by the expression:

$$\alpha_0 = \arcsin\frac{1}{\sqrt{F_0}}.$$



To determine the initial abscissa of the flow

Next, we will prove that $X_D^I \propto h_0/b$ and $X_D^I \propto X^*$ [1]. It's obvious that:

$$X_D^I \propto \frac{1}{\sin\theta_{max}}.$$

Consider the fall time of the water and the time of its inertial motion. Let h_0 = 10 cm, V_0 = 100 cm/s, X_T^I = 4 cm, then the free fall time of a liquid particle from a height of h_0 will take the form:

$$t_f = \sqrt{\frac{2h_0}{g}} = \sqrt{\frac{2\cdot 10}{1{,}000}} = \sqrt{\frac{1}{50}} = 0.145 \text{ s},$$

and the time of inertia travel $t_i = X_D/V_0 = 4/100 = 0.04$ s. The inertia time of a water particle is 4 times shorter than its fall time. Thus, the water, before it has time to fly away, comes out of the pipe as a jet. Therefore, the greater the depth of the flow, the greater the length of the inertial front, i.e. $X_T^I \propto h_0/b$, and $X_T^I \propto h_0/H_0$.

From the triangle ABC can be written (see Figure):

$$X_D^I \propto X^* = \frac{b}{2\,\text{tg}\alpha_0}, \quad \sin\alpha_0 = \frac{1}{\sqrt{F_0}}, \quad \text{tg}\alpha_0 = \frac{1}{\sqrt{F_0 - 1}},$$

then [3]:

$$X^* = \frac{b\sqrt{F_0 - 1}}{2}.$$

Let the initial abscissa of the flow X* be directly proportional to the length of the inertial front X_D^I, and also directly proportional to the flow depth h_0 and inversely proportional to $\sin\theta_{max}$:

$$X_D^I \propto \frac{b\sqrt{F_0 - 1}}{2}, \; X_D^I \propto \frac{h_0}{H_0},$$

$$X_D^I \propto \frac{h_0}{b}, \; X_D^I \propto \frac{1}{\sin\theta_{max}},$$

we get the structure of the formula:

$$X_D^I \approx \frac{b\sqrt{F_0 - 1}}{2}\frac{1}{\sin\theta_{max}} \times$$

$$\times \frac{h_0}{b}\frac{h_0}{H_0} \approx h_0\frac{\sqrt{F_0 - 1}}{2}\frac{1}{\sin\theta_{max}}\frac{2}{(F_0 + 2)}.$$

Whence can be written:

$$\frac{X_D^I}{h_0} \propto \frac{\sqrt{F_0 - 1}}{F_0 + 2}\frac{1}{\sin\theta_{max}}.$$

As a result of regression analysis applied to the experimental data, the authors obtained a formula for the length of the inertial front of the flow when it spreads from a rectangular tube into a wide diversionary channel in the form of:

$$X_D^I = \text{trunc}\left[\frac{\sqrt{F_0 - 1}}{F_0 + 2}\frac{1}{\sin\theta_{max}}h_0\right] + 1,$$

[1] The sign ∝ denotes the proportionality of the quantities.

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which is verified by experiment. Here "1" is taken in length units: for the experiment, it is a centimeter, for nature calculations it is a meter.

Determination the distribution law of the depth and velocity along its longitudinal symmetry axis. The dynamic equations of motion of a two-dimensional open flow are obtained from the equations of L. Euler, supplementing them with terms that take into account the resistance forces. Complications also arise when taking into account the slope of the bottom of the conduit, i.e. the need to take into account the influence of gravity. As a result, the system of equations of motion and continuity of the flow form a system of essentially non-linear partial differential equations, the analytical solution of which has not been found so far.

The study of the flow equations begins with the simplest case.

Where in:

• resistance forces are neglected (it is true for supercritical flows passing through a narrowing, or an expansion, or a bend in the channel, since in these cases, inertial forces have a predominant effect on the flow);

• movement is considered potential;

• the bottom of the flow is assumed to be horizontal.

In this case, the equations of motion of a two-dimensional supercritical flow turn out to be similar to the equations of plane gas motion at supersonic speeds. Therefore, many known results of gas dynamics can be applied to supercritical flows [2]. Let's use the known method of S.A. Chaplygin to move to the velocity hodograph plane, that is, we pass from the coordinates in the plane to the variables of the velocity vector and its angle of inclination to the axis of symmetry of the flow. In this case, the nonlinear system of equations is reduced to a linear system of equations in partial derivatives, which greatly facilitates its study.

Using the velocity hodograph plane [2, 3], a differential complex relation between the flow plan and the velocity hodograph plane can be established in the form of a generalizing relationship between the conjugate velocity and the derivative of the complex potential along the coordinate [3]:

$$dz = dx + idy = \left(d\varphi + i\frac{h_0}{h}d\psi\right)\frac{1}{V}e^{i\theta}, \qquad (2)$$

where θ is the angle characterizing the direction of the velocity vector to the axis *Ox*; *V* is the modulus of the liquid particle velocity vector.

The main system of planned equations of motion for stationary, potential water flows without taking into account the internal friction of the flow and its friction against the channel has the form[2, 3] [2, 3, 22]:

$$\begin{cases} \dfrac{\partial\varphi}{\partial\tau} = -\dfrac{h_0}{2H_0}\dfrac{1-3\tau}{\tau(1-\tau)^2}\dfrac{\psi}{\partial\theta}, \\ \dfrac{\partial\varphi}{\partial\theta} = \dfrac{2h_0}{H_0}\dfrac{\tau}{1-\tau}\dfrac{\partial\psi}{\partial\tau}. \end{cases} \qquad (3)$$

One of the solutions to system (3) are the stream functions and the potential function of the form:

$$\psi = \frac{A}{\tau^{1/2}}\sin\theta; \qquad \varphi = A\frac{h_0}{H_0}\frac{\cos\theta}{\tau^{1/2}(1-\tau)}. \qquad (4)$$

Assuming along the streamline dψ = 0, and taking into account that on the axis of symmetry of the flow θ = 0 taking into account (2) and (4), we obtain an ordinary differential equation relating dx, $d\tau$:

$$dx = \frac{Ah_0}{2H_0\sqrt{2gH_0}}\frac{(3\tau-1)}{\tau^2(1-\tau)^2}d\tau, \qquad (5)$$

where indicated $A = V_0 b/2\sin\theta_{max}$ — constant for the entire flow; τ is the parameter of the flow kinetics.

Integration of equation (5) taking into account the initial conditions $x = X_D^I + X^*$, $\tau = \tau_0$ allows you to get a dependency of the form:

$$x = X_D^I + X^* + \frac{Ah_0}{2H_0\sqrt{2gH_0}}\times \left[\frac{1+\tau}{\tau(1-\tau)} - \ln\frac{1-\tau}{\tau} - \frac{1+\tau_0}{\tau_0(1-\tau_0)} + \ln\frac{1-\tau_0}{\tau_0}\right]. \qquad (6)$$

The values of the flow velocity and its depth are found by the formulas [6]:

$$V = \tau^{1/2}\sqrt{2H_0 g}; \qquad h_0 = H_0(1-\tau). \qquad (7)$$

The following problems [22, 23] can be solved using the formula (6).

Problem 1. By setting the kinetics parameter on the flow symmetry axis, the abscissa of this point on the symmetry axis, counting from the edge of the pipe, can be determined, then the depth at this point and the flow velocity can be found.

Problem 2. By setting the abscissa of a point on the longitudinal axis of flow symmetry, the kinetic

[2] Certificate of state registration of computer programs No. 2022618552. Determination of Parameters of a Freely Spreading Flow. *Burtseva O.A. et al.* 05.12.2022.

[3] Certificate of state registration of computer programs No. 2022666655. Determination of Flow Parameters Along the Extreme Current Line. Aleksandrova M.S. 08.29.2022.

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parameter can be found from equation (5), and further, the flow velocity and depth at that point can be determined.

Problem 3. By setting the flow depth, it is possible to determine the flow velocity, the kinetics parameter, and then from equation (5) the abscissa of this point on the symmetry axis, counting from the pipe edge.

The set tasks are implemented as separate blocks in MathCad environment[3].

Specifying the distribution law of the flow parameters along the outermost current line. To determine the coordinates of the outermost current line, the coupling equation between the physical flow plane and the velocity hodograph plane (2) is used.

For the current and potential functions from equation (2), taking into account solution (4), we obtain a system of differential equations that is valid along the outermost flow line [3]:

$$\begin{cases} dx = -\dfrac{Ah_0 \cos\theta}{2H_0\tau^{1/2}\sqrt{2gH_0}} \times \\ \times\left[\dfrac{(1-3\tau)}{2\tau(1-\tau)^2}\dfrac{\cos\theta}{\tau^{1/2}}d\tau + \dfrac{\tau}{1-\tau}\dfrac{\sin\theta}{\tau^{3/2}}d\theta\right], \\ dy = -\dfrac{Ah_0 \sin\theta}{2H_0\tau^{1/2}\sqrt{2gH_0}} \times \\ \times\left[\dfrac{(1-3\tau)}{2\tau(1-\tau)^2}\dfrac{\cos\theta}{\tau^{1/2}}d\tau + \dfrac{\tau}{1-\tau}\dfrac{\sin\theta}{\tau^{3/2}}d\theta\right]. \end{cases} \quad (8)$$

A long the outermost flow line the dependence is valid [3, 12]:

$$\frac{\sin\theta}{\tau^{1/2}} = \sin\theta_{\max}. \quad (9)$$

Since along the outermost flow line $d\psi = 0$, then from the first equation of the system (4) we obtain the connection between the parameters:

$$\cos\theta d\theta = 1/2\sin\theta\frac{d\tau}{\tau}. \quad (10)$$

Equation (10), taking into account dependence (9), is reduced to the form:

$$\cos\theta d\theta = 1/2\frac{d\tau}{\tau^{1/2}}\sin\theta_{\max}.$$

Taking into account the transformations made above, we transform the system of equations (7) to the form:

$$\begin{cases} dx = \dfrac{Ah_0}{2H_0\sqrt{2gH_0}}\left[\dfrac{3\tau-1}{\tau^2(1-\tau)^2} - \dfrac{2\sin^2\theta_{\max}}{(1-\tau)^2}\right]d\tau, \\ dy = \dfrac{Ah_0}{H_0\sqrt{2gH_0}}d\left[\dfrac{\cos\theta}{\tau^{1/2}(1-\tau)}\right]. \end{cases} \quad (11)$$

Integration of system (11) allows one to obtain the parametric equations of the outermost flow line in the form [3]:

$$\begin{cases} x = X_D^I + \dfrac{Ah_0}{2H_0\sqrt{2gH_0}}\left[\dfrac{1+\tau}{\tau(1-\tau)} - \dfrac{2\sin^2\theta_{\max}}{1-\tau} - \right. \\ \left. -\ln\dfrac{1-\tau}{\tau} - \dfrac{1+\tau_k}{\tau_k(1-\tau_k)} + \dfrac{2\sin^2\theta_{\max}}{1-\tau_k} + \ln\dfrac{1-\tau_k}{\tau_k}\right]d\tau, \\ y = \dfrac{b}{2} + \dfrac{Ah_0\sin^2\theta_{\max}}{H_0\sqrt{2gH_0}}\left[\dfrac{\cos\theta}{\tau^{1/2}(1-\tau)} - \dfrac{\cos\theta_k}{\tau_k^{1/2}(1-\tau_k)}\right]. \end{cases} \quad (12)$$

System of equations (12) at known parameters θ_k and τ_k [3] at the corner point of the outlet edge pipe of the outermost flow line allows to determine the coordinates of the very point X, Y of an arbitrary point of the outermost flow line.

Using the system of equations (12) the following problems can be solved.

Problem 4. On the equipotentiality, the kinetic flow parameters and the angle of inclination of the flow velocity vector to the longitudinal axis of symmetry have the same value. By setting the parameter τ_w at the point W on the flow symmetry axis, it is possible to determine the parameters τ_C, θ_C of the point C at the outermost flow line and then the coordinates x_C, y_C of this point. And further changing the parameter τ_W, we obtain a set of points outermost flow line.

Problem 5. Assuming in the first equation of the system (12) $x = x_C$, we determine the kinetic parameter τ_C at the point C of the flow at the outermost flow line and then the angle of the velocity vector to the longitudinal axis of symmetry of the flow θ_C at this point. Substituting the found flux kineticity into the second equation of the system (12), we determine the ordinate of the outermost flow line y_C.

Thus, given a flow extension $\beta_C = V_C/b_0$ it is possible to estimate the distance mismatch for a selected flow cross-section.

The authors of this study are actively pursuing research in this area, improving and refining the proposed algorithms.

RESULTS

Let us consider the tasks. Experimental data on the free spreading of the flow behind a rectangular culvert are borrowed from [3], where the experimental setup, flow measuring instruments, other details of the experiment are described in detail, see Table 1. The article presents only the results of a numerical calculation using the developed programs[2, 3].

The flow has the following characteristics:
- initial flow velocity V_0 = 147.654 cm/s;
- initial flow depth relative to the bottom h_0 = 9.27 cm;
- acceleration of gravity g = 981 cm/s^2;
- pipe width b = 16 cm.

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Table 1. Point characteristics of free spreading of the plan flow

X_E, cm	4	24	44	64	71
Y_E, cm	9.5	38	59	76	80
h, cm	8.5	2.66	1.54	1.12	1.09
V, cm/s	151.928	186.461	191.243	192.714	191.49

Next we find:

- Froude's number $F_0 = 2.397$;
- hydrodynamic pressure $H_0 = 20.382$ cm;
- initial flow kinetics $\tau_0 = 0.545$;
- inertial front length $X_D^I = 3$ cm;
- wave angle at a point M_0 $\alpha_0 = 0.702$ r or $\alpha_0 = 40°23'$.

1. Let us set a kinetic parameter on flow symmetry axis, dividing segment $[\tau_0, 1]$ into equal parts in increments $\Delta\tau = 4.548 \cdot 10^{-3}$. Then we calculate the abscissa on the symmetry axis corresponding to the value of kinetic flow according to formula (6). Then we determine flow velocities and depths at the corresponding point. The results are given in Table 2. For the sake of convenience, the data for points where abscissa are close to experimental values are given.

Table 2. Determination of the abscissa on the longitudinal axis of the flow according to a given value of the kinetic parameter

Step number	τ_i	X_i, cm	V_i, cm/s	h_i, cm
70	0.864	24.477	185.831	2.772
83	0.923	43.493	192.087	1.569
89	0.95	66.978	194.907	1.121
90	0.955	73.612	195.373	1.019

2. In order to determine the kinetic parameter of the flow, let us set an abscissa on its longitudinal axis with the least deviation with experimental data, for instance 4 cm step. Solving equation (6), we find the only root corresponding to the kinetic parameter at the point on the longitudinal axis of the stream with given abscissa. Then we find the velocity and the depth of the flow at the given point. The results of the problem solution are given in Table 3.

3. The depth of flow h we set, using the calculations in problem 2. The kinetic parameter is determined from formula (7). Then we find abscissa on flow symmetry axis corresponding to this kinetic parameter from equation (6), then find flow velocity. The results of the problem solution are given in Table 4. For the convenience of analysis, the data at the same points as in Table 3 are given.

Table 3. Determination of the kinetic parameter for a given abscissa

Step number	τ_i	X_i, cm	V_i, cm/s	h_i, cm
6	0.861	24	185.548	2.834
11	0.924	44	192.179	1.558
16	0.948	64	194.664	1.068
18	0.953	72	195.267	0.948

It should be noted that there is no discrepancy between the calculated data in Table 3 and Table 4. This is due to the fact that the calculation is performed using analytical relationships.

Table 4. Definitions of the kinetic parameter, point abscissa and flow velocity on the axis of symmetry for a given flow depth

Step number	τ_i	X_i, cm	V_i, cm/s	h_i, cm
6	0.861	24	185.548	2.834
11	0.924	44	192.179	1.558
16	0.948	64	194.664	1.068
18	0.953	72	195.267	0.948

4. Now let us consider the determination of flow parameters along the outermost flow line. From the second equation of system (4) it is seen that an equipotential can be singled out by a specific value of the parameter τ on the flow symmetry axis. By setting the parameter τ_0 for the point M_0 (Figure) on the flow symmetry axis, we determine the parameters τ_C, θ_C followed by the coordinates x_C, y_C of the point C on the outermost flow line corresponding to this equipotentiality. Then by changing the kinetic parameter (e.g. with a constant step) we obtain the set of points of the outermost flow line.

5. Assuming in the first equation of the system (12) $x = x_C$, we determine the kinetic parameter τ_C at the point C of the flow at the outermost flow line and then the angle of the velocity vector to the longitudinal axis of symmetry of the flow θ_C at this point. Substituting the found flux kineticity into the second equation of the system (12), let us determine the ordinate of the outermost flow line y_C. The results coincide with the numerical experiment described in the previous paragraph.

The numerical experiment consisted in calculating the values of the kinetic parameter, the angle of inclination of the flow velocity vector to the symmetry axis and the coordinates of the points on the outermost flow line. The results are given in Table 5. In order to determine the adequacy of the parameter calculation algorithm, a relative error of the flow ordinate has been calculated in comparison with experimental data.

CONCLUSIONS

The formation of a mathematical model of water flow is based on its division into several sections. At the stage of flow exit from the pipe there is a section where flow parameters (velocity, depth, width) are kept constant — inertial front. The formula of the inertial front length of the water stream has been derived at its spread from a rectangular pipe into a wide outlet

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Table 5. Definitions of the kinetic parameter, the angle of inclination of the flow velocity vector to the axis of symmetry, and the ordinate of points on the extreme streamline according to the given kinetic parameters on the axis of symmetry of the flow

Step number	X_i, cm	τ_i on the axis of symmetry	τ_{Ci} on outermost flow line	θ_{Ci}	Y_{Ci}, cm/s	$\frac{\lvert Y_{Ci} - Y_e \rvert}{Y_{Ci}} 100\ \%$
0	0	0.545	0.545	0.661	8	0
1	4	0.545	0.731	0.791	8.97	5.914
6	24	0.861	0.952	0.946	34.926	9.854
11	44	0.924	0.975	0.963	63.3	6.241
16	64	0.948	0.983	0.969	92.257	17.275
18	72	0.953	0.985	0.97	103.922	22.729

channel by introducing dimensionless complexes and on the base of π-theorem.

Obtained formula for the length of the inertial front, allows us to achieve the desired error. At flow extensions up to 5, relative errors of ordinates and flow velocities do not exceed 7–10 %. At steps 16 and 18 the error is more than 10 %. This can be explained by the fact that these points are close to the transition zone. This area should be investigated further.

Using analogies from gas dynamics, namely the transition to the flow velocity hodograph plane, it is possible to obtain the depth and velocity distribution laws along the longitudinal axis of symmetry and along the outermost flow line. With the help of system of equations (12) set tasks have been successfully solved. Software has been developed for realization of these algorithms. On the basis of experimental data, the adequacy of the refined mathematical model of two-dimensional planar flow has been proved.

REFERENCES

1. Sherenkov I.A. Experimental studies of the spreading of a turbulent flow behind the outlet heads of culverts. *Proceedings of the joint seminar on hydraulic engineering and water management construction*. Kharkiv, 1958; 1:39-43. (rus.).

2. Emcev B.T. *Two-dimensional turbulent flows*. Moscow, Energiya Publ., 1967; 212. (rus.).

3. Kokhanenko V.N., Volosukhin Ya.V., Lemeshko M.A., Papchenko N.G. *Modeling of stormy two-dimensional in plane water flows*. Rostov-on-Don, Southern Federal University Publ., 2013; 180. (rus.).

4. Konstantinov N.M. Some issues of the hydraulics of the tailwater of small road culverts with free spreading of a turbulent flow. *Hydraulics of road culverts*. 1969; 255-269. (rus.).

5. Kosichenko N.V. About the petal of the free flow of a turbulent stream into a wide fortified channel. *Prirodoobustrojstvo (Environmental Engineering)*. 2011; 3:58-62. URL: https://cyberleninka.ru/article/n/o-lepestke-svobodnogo-rastekaniya-burnogo-potoka-v-shirokoe-ukreplennoe-ruslo (rus.).

6. Kosichenko N.V. Analysis of the study and refinement of methods for free flow spreading behind non-pressure culverts. *Bulletin of the Saratov State Agrarian University*. 2011; 9:27-33. (rus.).

7. Kohanenko V.N., Mitsik M.F., Kosichenko N.V. The more accurate equation of the extreme line current in the hodograph plane speed in free flow stormy spreading for unpressurised pipes. *Bulletin of Higher Educational Institutions. North Caucasus region. Technical Sciences*. 2013; 1:33-35. URL: https://cyberleninka.ru/article/n/utochnennoe-uravnenie-krayney-linii-toka-v-ploskosti-godografa-skorosti-v-zadache-svobodnogo-rastekaniya-burnogo-potoka-za-beznapornymi (rus.).

8. Micik M.F., Kohanenko N.V., Kosichenko N.V. Determination of parameters of a turbulent flow, freely spreading from a pressureless tube in a broad channel without the flooding of the lower canal pond. *Privolzhsky Scientific Journal*. 2013; 4:52-56. URL: https://www.elibrary.ru/item.asp?id=21306122 (rus.).

9. Tang S.L., Antonia R.A., Djenidi L., Abe H., Zhou T., Danaila L., Zhou Y. Transport equation for the mean turbulent energy dissipation rate on the centreline of a fully developed channel flow. *Journal of Fluid Mechanics*. 2015; 777:151-177. DOI: 10.1017/jfm.2015.342

10. Anderson W., Barros J.M., Christensen K.T., Awasthi A. Numerical andexperimental study of mechanisms responsible for turbulent secondary flows in boundary layerflows over spanwise heterogeneous roughness. *Journal of Fluid Mechanics*. 2015; 768:316-347. DOI: 10.1017/jfm.2015.91

11. Aranda J.Á., Beneyto C., Sánchez-Juny M., Bladé E. Efficient design of road drainage systems. *Water*. 2021; 13:1661. DOI.org/10.3390/w13121661.

12. Anees M.T., Abdullah K., Nordin M.N., Rahman N.N., Syakir M.I., Kadir M.O. One- and two-dimensional hydrological modelling and their uncertain-

ties. *Flood Risk Management*. 2017; 11:221-244. DOI: 10.5772/intechopen.68924.

13. Nematollahi B., Abedini M.J. Analytical solution of gradually varied flow equation in non-prismatic channels. *Iranian Journal of Science and Technology — Transactions of Civil Engineering*. 2020; 44(1):251-258. DOI: 10.1007/s40996-019-00316-5

14. Hager W., Castro-Orgaz O. Transcritical flow in open channel hydraulics: from böss to de marchi. *Journal of Hydraulic Engineering*. 2016; 142(1). DOI: 10.1061/(asce)hy.1943-7900.0001091

15. Hager W. Unconfined expansion of supercritical water flow. *Journal of Engineering Mechanics*. 1997; 123(5):451-457. DOI: 10.1061/(ASCE)0733-9399(1997)123:5(451)

16. Liu J.L., Wang Z.Z., Zhao Y.F. Explicit equations for critical depth in open channels with complex compound cross sections. *Flow Measurement and Instrumentation*. 2012; 24:13-18. DOI: 10.1016/j.flowmeasinst.2011.12.005

17. Chanson H. Explicit equations for critical depth in open channels with complex compound cross sections: A discussion. *Flow Measurement and Instrumentation*. 2013; 29:65-66. DOI: 10.1016/j.flowmeasinst.2012.10.006

18. Castro-Orgaz O., Cantero-Chinchilla F.N. Non-linear shallow water flow modelling over topography with depth-averaged potential equations. *Environmental Fluid Mechanics*. 2020; 20(2):261-291. DOI: 10.1007/s10652-019-09691-z

19. Li J., Li S.S. Near-bed velocity and shear stress of open-channel flow over surface roughness. *Environmental Fluid Mechanics*. 2020; 20(2):293-320. DOI: 10.1007/s10652-019-09728-3

20. Jesusdhas V., Balachandar R., Wang H., Murzyn F. Modelling hydraulic jumps: IDDES versus experiments. *Environmental Fluid Mechanics*. 2020; 20(2):393-413. DOI: 10.1007/s10652-019-09734-5

21. Leng X., Chanson H. Hybrid modelling of low velocity zones in box culverts to assist upstream fish passage. *Environmental Fluid Mechanics*. 2020; 20(2):415-432. DOI: 10.1007/s10652-019-09700-1

22. Kokhanenko V., Burtseva O., Kelekhsaev D. Here determination of the free flow maximum expansion into a wide diversion channel of turbulent water flow behind the non-pressure water throughput hole. *IOP Conference Series: Materials Science and Engineering*. 2020; 753(3):032053. DOI: 10.1088/1757-899X/753/3/032053

23. Kohanenko V.N., Kelekhsaev D.B. Solution of the problem of determining the equation of the extreme streamline and the parameters along it taking into account the section XD. *Research Results — 2019: materials of the IV National Conference of the Faculty and Researchers (Novocherkassk, May 14, 2019)*. Novocherkassk, 113-117. (rus.).

24. Sedov L.I. *Methods of similarity and dimension in mechanics*. Moscow, Nauka Publ., 1977; 440. (rus.).

25. Romanov A.S., Semikolenov A.V., Taranenko S.N., Shahorin A.P. *Theory of similarity and dimension. Boundary layer*. Moskva, MGTU im. N.E. Baumana, 2011; 48. (rus.).

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СПИСОК ИСТОЧНИКОВ

1. *Шеренков И.А.* Экспериментальные исследования растекания бурного потока за выходными оголовками водопропускных сооружений // Труды объединенного семинара по гидротехническому и водохозяйственному строительству. Харьков, 1958. Вып. 1. С. 39–43.

2. *Емцев Б.Т.* Двухмерные бурные потоки. М. : Энергия, 1967. 212 с.

3. *Коханенко В.Н., Волосухин Я.В., Лемешко М.А., Папченко Н.Г.* Моделирование бурных двухмерных в плане водных потоков. Ростов н/Д : Издательство Южного федерального университета, 2013. 180 с.

4. *Константинов Н.М.* Некоторые вопросы гидравлики нижнего бьефа малых дорожных водопропускных сооружений при свободном растекании бурного потока // Гидравлика дорожных водопропускных сооружений. 1969. С. 255–269.

5. *Косиченко Н.В.* О лепестке свободного растекания бурного потока в широкое укрепленное русло // Природообустройство. 2011. № 3. С. 58–62. URL: https://cyberleninka.ru/article/n/o-lepestke-svobodnogo-rastekaniya-burnogo-potoka-v-shirokoe-ukreplennoe-ruslo

6. *Косиченко Н.В.* Анализ изучения и уточнения методов свободного растекания потока за безнапорными водопропускными отверстиями // Вестник Саратовского ГАУ. 2011. № 9. С. 27–33.

7. *Коханенко В.Н., Мицик М.Ф., Косиченко Н.В.* Уточненное уравнение крайней линии тока в плоскости годографа скорости в задаче свободного растекания бурного потока за безнапорными водопропускными трубами // Известия вузов. Северо-Кавказский регион. Технические науки. 2013. № 1. С. 33–35. URL: https://cyberleninka.ru/article/n/utochnennoe-uravnenie-krayney-linii-toka-v-ploskosti-godografa-skorosti-v-zadache-svobodnogo-rastekaniya-burnogo-potoka-za-beznapornymi

8. *Мицик М.Ф., Коханенко Н.В. Косиченко Н.В.* Определение параметров свободно растекающегося бурного потока в окрестности его выхода из безнапорной прямоугольной трубы в широкое отводящее русло // Приволжский научный журнал. 2013. № 4. С. 52–56. URL: https://www.elibrary.ru/item.asp?id=21306122

9. *Tang S.L., Antonia R.A., Djenidi L., Abe H., Zhou T., Danaila L., Zhou Y.* Transport equation for the mean turbulent energy dissipation rate on the centre-line of a fully developed channel flow // Journal of Fluid Mechanics. 2015. Vol. 777. Pp. 151–177. DOI: 10.1017/jfm.2015.342

10. *Anderson W., Barros J.M., Christensen K.T., Awasthi A.* Numerical andexperimental study of mechanisms responsible for turbulent secondary flows in boundary layerflows over spanwise heterogeneous roughness // Journal of Fluid Mechanics. 2015; 768:316-347. DOI: 10.1017/jfm.2015.91

11. *Aranda J.Á., Beneyto C., Sánchez-Juny M., Bladé E.* Efficient design of road drainage systems // Water. 2021. Vol. 13. P. 1661. DOI: 10.3390/w13121661

12. *Anees M.T., Abdullah K., Nordin M.N., Rahman N.N., Syakir M.I., Kadir M.O.* One- and two-dimensional hydrological modelling and their uncertainties // Flood Risk Management. 2017. Vol. 11. Pp. 221–244. DOI: 10.5772/intechopen.68924

13. *Nematollahi B., Abedini M.J.* Analytical solution of gradually varied flow equation in non-prismatic channels // Iranian Journal of Science and Technology — Transactions of Civil Engineering. 2020. Vol. 44. Issue 1. Pp. 251–258. DOI: 10.1007/s40996-019-00316-5

14. *Hager W., Castro-Orgaz O.* Transcritical flow in open channel hydraulics: from böss to de marchi // Journal of Hydraulic Engineering. 2016. Vol. 142. Issue 1. DOI: 10.1061/(asce)hy.1943-7900.0001091

15. *Hager W.* Unconfined expansion of supercritical water flow // Journal of Engineering Mechanics. 1997. Vol. 123. Issue 5. Pp. 451–457. DOI: 10.1061/(ASCE)0733-9399(1997)123:5(451)

16. *Liu J.L., Wang Z.Z., Leng C.J., Zhao Y.F.* Explicit equations for critical depth in open channels with complex compound cross sections // Flow Measurement and Instrumentation. 2012. Vol. 24. Pp. 13–18. DOI: 10.1016/j.flowmeasinst.2011.12.005

17. *Chanson H.* Explicit equations for critical depth in open channels with complex compound cross sections: A discussion // Flow Measurement and Instrumentation. 2013. Vol. 29. Pp. 65–66. DOI: 10.1016/j.flowmeasinst.2012.10.006

18. *Castro-Orgaz O., Cantero-Chinchilla F.N.* Non-linear shallow water flow modelling over topography with depth-averaged potential equations // Environmental Fluid Mechanics. 2020. Vol. 20. Issue 2. Pp. 261–291. DOI: 10.1007/s10652-019-09691-z

19. *Li J., Li S.S.* Near-bed velocity and shear stress of open-channel flow over surface roughness // Environmental Fluid Mechanics. 2020. Vol. 20. Issue 2. Pp. 293–320. DOI: 10.1007/s10652-019-09728-3

20. *Jesusdhas V., Balachandar R., Wang H., Murzyn F.* Modelling hydraulic jumps: IDDES versus experiments // Environmental Fluid Mechanics. 2020. Vol. 20. Issue 2. Pp. 393–413. DOI: 10.1007/s10652-019-09734-5

21. *Leng X., Chanson H.* Hybrid modelling of low velocity zones in box culverts to assist upstream fish passage // Environmental Fluid Mechanics. 2020. Vol. 20. Issue 2. Pp. 415–432. DOI: 10.1007/s10652-019-09700-1

22. *Kokhanenko V., Burtseva O., Kelekhsaev D.* Here determination of the free flow maximum expansion into a wide diversion channel of turbulent water flow behind the non-pressure water throughput hole // IOP Conference Series: Materials Science and Engineering. 2020. Vol. 753. Issue 3. P. 032053. DOI: 10.1088/1757-899X/753/3/032053

23. *Коханенко В.Н., Келехсаев Д.Б.* Решение задачи определения уравнения крайней линии тока и параметров вдоль нее с учетом участка ХД // Результаты исследований — 2019 : мат. IV Национальной конф. проф.-преп. состава и научных работников

(г. Новочеркасск, 14 мая 2019 г.). Новочеркасск, 2019. С. 113–117.

24. *Седов Л.И.* Методы подобия и размерности в механике. М. : Наука, 1977. 440 с.

25. *Романов А.С., Семиколенов А.В., Тараненко С.Н., Шахорин А.П.* Теория подобия и размерности. Пограничный слой. М. : МГТУ им. Н.Э. Баумана, 2011. 48 с.

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